APPENDIX

Here we show that CATA guarantee 50% optimality when local reward function satisfies the *diminishing marginal gain* condition.

Proof: Each round of auction produces one globally highest bid. For notational convenience, we use the same symbol for both the round identifier and the ID of the robot that wins the auction at the corresponding round. In other words, robot r_i won the auction at round i with bid $b_{gi,i}$ and robot r_j won the auction at round j with bid $b_{gj,j}$. We assume $i < j$ for the rest of this section.

Because only the globally highest bid wins the auction, the following condition holds:

$$b_{gi,i} \geq b_{jA(r_i),i} \quad \forall i, j \in 1, ...N_R \tag{9}$$

where A is an assignment set that can be searched with either the robot ID or the task identifier, $A(r_i)$ gives the task that robot r_i has won at round *i*. Because each robot only submits its local highest bid and its local reward function satisfies the *diminishing marginal gain* condition, which means the bid that any robot can submit for any task monotonically decreases as the auction proceeds, the following condition holds:

$$b_{gi,i} \geq b_{il,i} \geq b_{il,j} \quad \forall i \in 1, ...N_R \;\; \forall l \in \tag{10}$$

Consider swapping the tasks of robot r_i and r_j, their combined bid changes from $b_{gi,i}+b_{gj,j}$ to $b_{iA(r_j),j}+b_{jA(r_i),i}$. Because the inequalities in (9) and (10) hold, the new combined bid is upper bounded as below.

$$b_{iA(r_j),j} + b_{jA(r_i),i} \leq b_{gi,i} + b_{gi,i} = 2b_{gi,i} \tag{11}$$

And the largest improvement of the combined bid of robot r_i and r_j is achieved when

$$b_{iA(r_j),j} = b_{jA(r_i),i} = b_{gi,i} \tag{12}$$

Now consider CATA provides a solution so that the objective value is

$$CATA = \sum_{i=1}^{N} b_{gi,i} \tag{13}$$

where N refers to the maximum number of tasks that can be assigned. Since each robot can only take one task, any possible variation of the assignment should be in the form of a sequence of task swapping. The largest possible improvement of the objective value can be achieved after a specific sequence of task swapping while every task swapping satisfies the condition (12). Therefore the optimal objective value (OOV) should satisfy

$$OOV = \sum_{i=1}^{N_{swapped}/2} b_{iA(r_j),j} + \sum_{j=1}^{N_{swapped}/2} b_{jA(r_i),i} + \sum_{i=1}^{N_{unswappped}} b_{gi,i}$$

$$= 2 \sum_{i=1}^{N_{swapped}/2} b_{gi,i} + \sum_{i=1}^{N_{unswapped}} b_{gi,i}$$

$$\leq 2 \sum_{i=1}^{N} b_{gi,i} = 2CATA$$

where $N_{swapped}$ refers to the number of tasks that need to be swapped to achieve the largest possible improvement, and $N_{unswapped}$ refers to the rest of the assignments.

Thus, $CATA \geq OOV/2$, the 50% optimality is guaranteed.

ACKNOWLEDGMENT

We would like to thank NSERC for supporting this work under the NSERC Strategic Partnership Grant (479149). Simulations were performed using the computing clusters managed by Calcul Québec and Compute Canada.

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